

## EFFECT OF NUMBER OF KNOTS IN A POLYNOMIAL TRAJECTORY ON MOTION CHARACTERISTICS OF ROBOTIC MANIPULATORS

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### ABSTRACT

The paper presents a new method of robotic trajectory planning and motion characteristics. Main objective is to study the motion characteristics of a manipulator and to explore the scope of minimization of jerk. Multiple higher order polynomials are considered for the trajectory design and the effect of number of intermediate knots between start and final positions of a 3R manipulator within the workspace is studied. Values of displacements, velocities, accelerations and jerk of end-effectors on a straight line path are presented. There is an attempt to simulate the motion of the manipulator with the help of AutoLISP code on AutoCAD platform.

**Keywords:** Trajectory, Spline, Knots; Jerk, Manipulator.

### 1. INTRODUCTION

One common objective for using robotic manipulators in industries is to make them trace a specified trajectory. The trajectory planning is an active area of research in the field of robotics. N. A. Aspragathos [1] worked in the area of generation of Cartesian trajectory under bounded position deviation. Gasparetto and Zanotto [2] developed a new method for smooth trajectory planning. Saramago and Ceccarelli [3] proposed the optimization of trajectory planning taking into account robot actuating energy and grasping forces in manipulator gripper. In spite of many other similar works in the area of robotic trajectory planning there is a little number of papers published on the effects of a particular trajectory-curve on acceleration and more particularly on jerk of manipulators in motion.

This work is carried out to find the effect of number of knots in polynomial splines used as robotic trajectory on the motion characteristics of manipulator. Many works are there on effects of knots in polynomial spline on kinematics parameters like acceleration and jerk of cam followers [4]. Introduction of knots in polynomial spline and B-spline can play very important role in minimizing acceleration, jerk and ping (time derivative of jerk) of cam followers [5-6]. First study is conducted to see the effects of number of intermediate knots in 8<sup>th</sup> order polynomial on robotic trajectory [7]. With the help of these works higher order polynomial splines with multiple knots are designed for robotic trajectory. 6-, 8- and 10-order polynomials are considered and multiple knots are inserted to study the effects on trajectory design.

Different higher order polynomials are used to join

the knots in the joint space and polynomial splines are constructed. Effects of number of knots on displacement, velocity, acceleration and jerk for the linear path of end-effectors motion are analyzed. A case study is done with a 3R planar manipulator tracing the newly designed trajectory-curve. The simulation of the motion of the manipulator is done with the help of AutoLISP program on AutoCAD platform. The problem is approached in the following way:

(i) Start point and end point are assumed on a specified manipulator trajectory; (ii) the trajectory is defined in the coordinate space; (iii) some via-points (called knots) are assumed on the trajectory; (iv) inverse kinematics analysis at these points is done and the points are plotted in the joint coordinate system for each link of the robotic arm; (v) these points are joined by 6-, 8- and 10-order polynomials to construct splines; (vi) at each point forward kinematics analysis is done; (vii) simulation of motion of the manipulator is done in the workspace using AutoLISP code.

### 2. THEORETICAL ANALYSIS

Fig. 1 shows a 3R planar manipulator along with its work-space B and target trajectory ab, which is a straight line with two via-points p and q (called intermediate knots). B is a ring with inner and outer diameters. The number of intermediate knots can be as many as user's choice. Fig. 1(a) shows a spline with two end and intermediate knots. Inverse kinematics analysis at the points a, p, q, b is done and the points are plotted in the joint coordinate systems for each link - a and b being the

end knots. In a general way if there is a point P(x, y) in the coordinate space the inverse kinematics analysis requires to find the joint angles  $(\theta_1, \theta_2, \theta_3)$  as a function of wrist position and orientation  $(x, y, \phi)$  as shown in Fig. 2.

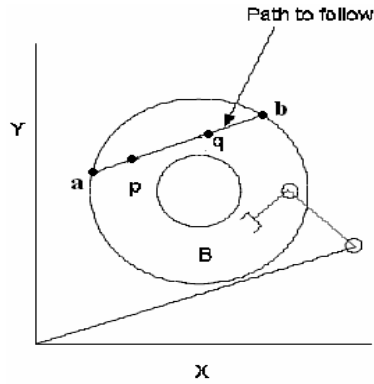


Fig 1. 3R planar manipulator: work-space and trajectory

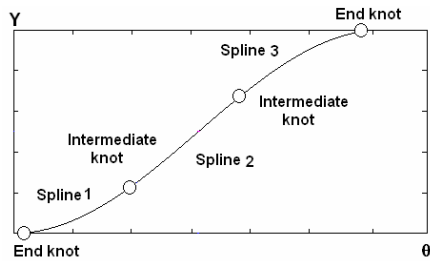


Fig 1(a). Spline with knots

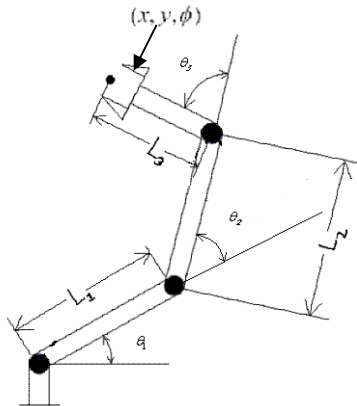


Fig 2. 3R planar manipulator: joint and link parameters  
Solving for  $\theta_1$  we rewrite the nonlinear using a change of variables as follows:

$$\begin{aligned} x &= L_1 c_1 + L_2 c_{12} \\ y &= L_1 s_1 + L_2 s_{12} \\ x &= k_1 c_1 + k_2 s_1 \\ y &= k_1 s_1 + k_2 c_1 \end{aligned}$$

where  $k_1 = L_1 + L_2 c_2$  and  $k_2 = L_2 s_2$  (Fig.3). Finally we compute  $\theta_2$  using the two argument arctangent function

$$\theta_1 = \text{atan2}(y, x) - \text{atan2}(k_2, k_1)$$

$$\theta_2 = \text{atan2}(s_2, c_2) = \text{atan2}\left(\pm \sqrt{1 - c_2^2}, \frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1 L_2}\right)$$

$$\theta_3 = \phi - \theta_1 - \theta_2$$

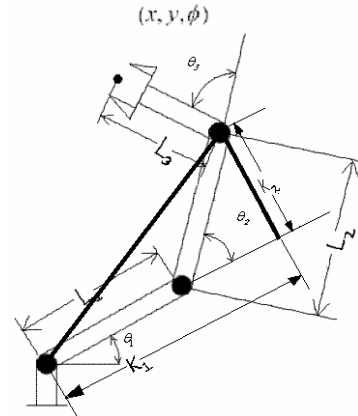


Fig 3.

These points are plotted in the joint co-ordinate system against time.  $\theta_1$  and  $\theta_2$  for a, p, q, b points are determined (as shown in fig 4 and fig 5).

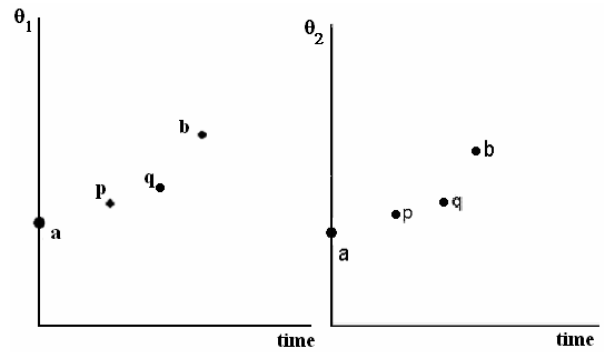


Fig 4.

Fig 5.

(Value of  $\theta_1, \theta_2$  at ends & intermediate positions of manipulator)

Here 6-, 8- and 10-order polynomials are considered for the sake of analysis. Calculation is shown for 8<sup>th</sup> order polynomial only. Velocity, acceleration and jerk are considered to be zero at end knots a and b. So we see position, velocity, acceleration and jerk at end knots are known, thereby, a total of 8 boundary conditions.

We plan to join the knots (a, p, q, b) by three 8<sup>th</sup> order polynomials (for k number of knots we use k-1 polynomials). In general if there are k number of knots and m order polynomial then number of unknown quantities:  $m(k-1)$ , smoothness equations:  $(k-2)(m-1)$ , interpolation equations:  $k-2$  and boundary conditions:  $m$  [4]. Calculations for 8<sup>th</sup> order polynomials connected by

4 knots are shown. For this above numbers will be 24, 14, 2 and 8 respectively. We consider here three polynomials to construct the polynomial spline. Eqs. (1-3) are the polynomials, Eqs. (4-17) the smoothness equations, Eqs. (18-19) the interpolation equations and Eqs. (20-27) the boundary condition equations.

Polynomials:

$$Y_1 = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + a_6 t^6 + a_7 t^7 \quad (1)$$

for  $0 < t < t_p$

$$Y_2 = b_0 + b_1 (t - t_p) + b_2 (t - t_p)^2 + b_3 (t - t_p)^3 + b_4 (t - t_p)^4 + b_5 (t - t_p)^5 + b_6 (t - t_p)^6 + b_7 (t - t_p)^7 \quad (2)$$

for  $t_p < t < t_q$

$$Y_3 = c_0 + c_1 (t - t_q) + c_2 (t - t_q)^2 + c_3 (t - t_q)^3 + c_4 (t - t_q)^4 + c_5 (t - t_q)^5 + c_6 (t - t_q)^6 + c_7 (t - t_q)^7 \quad (3)$$

for  $t_q < t < t_b$

Smoothness equations:

$$Y_1 = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + a_6 t^6 + a_7 t^7 \quad (4)$$

$$Y_1 = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4 + 6a_6 t^5 + 7a_7 t^6 \quad (5)$$

$$Y_1 = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3 + 30a_6 t^4 + 42a_7 t^5 \quad (6)$$

$$Y_1 = 6a_3 + 24a_4 t + 60a_5 t^2 + 120a_6 t^3 + 210a_7 t^4 \quad (7)$$

$$Y_1 = 24a_4 + 120a_5 t + 360a_6 t^2 + 840a_7 t^3 \quad (8)$$

$$Y_1 = 120a_5 + 720a_6 t + 2520a_7 t^2 \quad (9)$$

$$Y_1 = 720a_6 + 5040a_7 t \quad (10)$$

$$Y_2 = b_0 + b_1 (t - t_p) + b_2 (t - t_p)^2 + b_3 (t - t_p)^3 + b_4 (t - t_p)^4 + b_5 (t - t_p)^5 + b_6 (t - t_p)^6 + b_7 (t - t_p)^7 \quad (11)$$

$$Y_2 = b_1 + 2b_2 (t - t_p) + 3b_3 (t - t_p)^2 + 4b_4 (t - t_p)^3 + 5b_5 (t - t_p)^4 + 6b_6 (t - t_p)^5 + 7b_7 (t - t_p)^6 \quad (12)$$

$$Y_2 = 2b_2 + 6b_3 (t - t_p) + 12b_4 (t - t_p)^2 + 20b_5 (t - t_p)^3 + 30b_6 (t - t_p)^4 + 42b_7 (t - t_p)^5 \quad (13)$$

$$Y_2 = 6b_3 + 24b_4 (t - t_p) + 60b_5 (t - t_p)^2 + 120b_6 (t - t_p)^3 + 210b_7 (t - t_p)^4 \quad (14)$$

$$Y_2 = 24b_4 + 120b_5 (t - t_p) + 360b_6 (t - t_p)^2 + 840b_7 (t - t_p)^3 \quad (15)$$

$$Y_2 = 120b_5 + 720b_6 (t - t_p) + 2520b_7 (t - t_p)^2 \quad (16)$$

$$Y_2 = 720b_6 + 5040b_7 (t - t_p) \quad (17)$$

Interpolation equations:

$$b_0 = \theta_1 \quad \text{at } t = t_p \quad (18)$$

$$c_0 = \theta_1 \quad \text{at } t = t_q \quad (19)$$

Boundary conditions:

$$a_0 = Y_1 \quad (\text{at } t = 0) \quad (20)$$

$$a_1 = 0 \quad (\text{since at } t = 0, Y_1 = 0) \quad (21)$$

$$a_2 = 0 \quad (\text{since at } t = 0, Y_1 = 0) \quad (22)$$

$$a_3 = 0 \quad (\text{since at } t = 0, Y_1 = 0) \quad (23)$$

$$c_0 + c_1 (t_b - t_q) + c_2 (t_b - t_q)^2 + c_3 (t_b - t_q)^3 + c_4 (t_b - t_q)^4 + c_5 (t_b - t_q)^5 + c_6 (t_b - t_q)^6 + c_7 (t_b - t_q)^7 = Y_3 \quad (\text{at } t = t_b) \quad (24)$$

$$c_5 (t_b - t_q)^5 + c_6 (t_b - t_q)^6 + c_7 (t_b - t_q)^7 = Y_3 \quad (\text{at } t = t_b) \quad (25)$$

$$c_1 + 2c_2 (t_b - t_q) + 3c_3 (t_b - t_q)^2 + 4c_4 (t_b - t_q)^3 + 5c_5 (t_b - t_q)^4 + 6c_6 (t_b - t_q)^5 + 7c_7 (t_b - t_q)^6 = 0 \quad (26)$$

$$2c_2 + 6c_3 (t_b - t_q) + 12c_4 (t_b - t_q)^2 + 20c_5 (t_b - t_q)^3 + 30c_6 (t_b - t_q)^4 + 42c_7 (t_b - t_q)^5 = 0 \quad (27)$$

$$6c_3 + 24c_4 (t_b - t_q) + 60c_5 (t_b - t_q)^2 + 120c_6 (t_b - t_q)^3 + 210c_7 (t_b - t_q)^4 = 0 \quad (28)$$

$$6c_3 + 24c_4 (t_b - t_q) + 60c_5 (t_b - t_q)^2 + 120c_6 (t_b - t_q)^3 + 210c_7 (t_b - t_q)^4 = 0 \quad (29)$$

$$6c_3 + 24c_4 (t_b - t_q) + 60c_5 (t_b - t_q)^2 + 120c_6 (t_b - t_q)^3 + 210c_7 (t_b - t_q)^4 = 0 \quad (30)$$

$$6c_3 + 24c_4 (t_b - t_q) + 60c_5 (t_b - t_q)^2 + 120c_6 (t_b - t_q)^3 + 210c_7 (t_b - t_q)^4 = 0 \quad (31)$$

$$6c_3 + 24c_4 (t_b - t_q) + 60c_5 (t_b - t_q)^2 + 120c_6 (t_b - t_q)^3 + 210c_7 (t_b - t_q)^4 = 0 \quad (32)$$

$$6c_3 + 24c_4 (t_b - t_q) + 60c_5 (t_b - t_q)^2 + 120c_6 (t_b - t_q)^3 + 210c_7 (t_b - t_q)^4 = 0 \quad (33)$$

$$6c_3 + 24c_4 (t_b - t_q) + 60c_5 (t_b - t_q)^2 + 120c_6 (t_b - t_q)^3 + 210c_7 (t_b - t_q)^4 = 0 \quad (34)$$

$$6c_3 + 24c_4 (t_b - t_q) + 60c_5 (t_b - t_q)^2 + 120c_6 (t_b - t_q)^3 + 210c_7 (t_b - t_q)^4 = 0 \quad (35)$$

$$6c_3 + 24c_4 (t_b - t_q) + 60c_5 (t_b - t_q)^2 + 120c_6 (t_b - t_q)^3 + 210c_7 (t_b - t_q)^4 = 0 \quad (36)$$

$$6c_3 + 24c_4 (t_b - t_q) + 60c_5 (t_b - t_q)^2 + 120c_6 (t_b - t_q)^3 + 210c_7 (t_b - t_q)^4 = 0 \quad (37)$$

$$6c_3 + 24c_4 (t_b - t_q) + 60c_5 (t_b - t_q)^2 + 120c_6 (t_b - t_q)^3 + 210c_7 (t_b - t_q)^4 = 0 \quad (38)$$

$$6c_3 + 24c_4 (t_b - t_q) + 60c_5 (t_b - t_q)^2 + 120c_6 (t_b - t_q)^3 + 210c_7 (t_b - t_q)^4 = 0 \quad (39)$$

$$6c_3 + 24c_4 (t_b - t_q) + 60c_5 (t_b - t_q)^2 + 120c_6 (t_b - t_q)^3 + 210c_7 (t_b - t_q)^4 = 0 \quad (40)$$

$$6c_3 + 24c_4 (t_b - t_q) + 60c_5 (t_b - t_q)^2 + 120c_6 (t_b - t_q)^3 + 210c_7 (t_b - t_q)^4 = 0 \quad (41)$$

$$a_R = \sqrt{\dot{x}^2 + \dot{y}^2} \quad (32)$$

$$j_R = \sqrt{\ddot{x}^2 + \ddot{y}^2} \quad (33)$$

### 3. SIMULATION

Using AutoLISP code generated for the purpose we have simulated the motion of the manipulator arm tracing the newly designed trajectory in the coordinate space. Concepts of loop and file handling of AutoLISP were utilized for this. Drawing an entity and at the same time erasing the previous entity on the graphics screen are done rapidly at a speed selected by the user. This gives an effect of animation on the robotic manipulator on the graphics screen. AutoCAD Figs. (6-7) show simulation of manipulator motion in two positions. In course of simulation number of knots and order of the polynomial for trajectory planning has been varied and effect of these variations on velocity, acceleration and jerk is computed using MATLAB (R2009b).

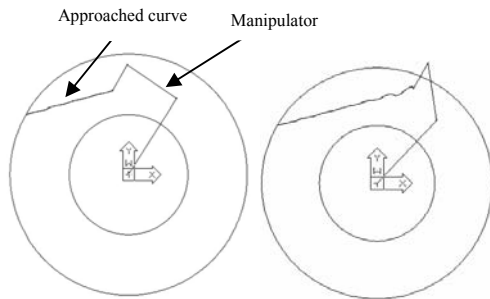


Fig 6. Simulation: 1

Fig 7. Simulation: 2

### 4. RESULTS

Results are shown in Figs. (8-16). Figs. (14-16) show the variations of maximum values of velocity, acceleration and jerk with the variation of number of knots and the order of the polynomial. Six cases are shown – for number of knots of 10 and 100 and polynomials of order 6,8 and 10.

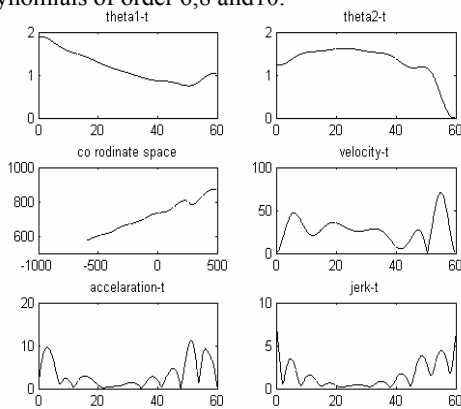


Fig 8. Polynomial order 6, number of knots10

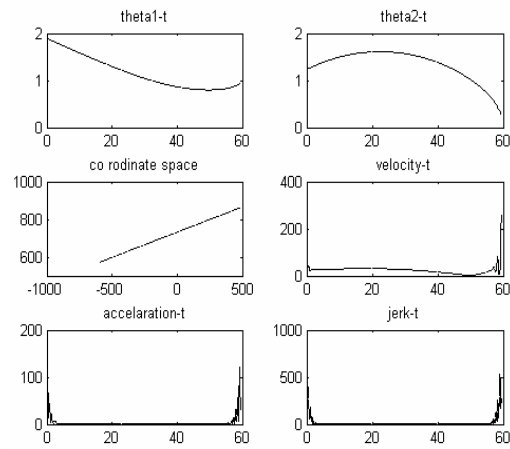


Fig 9. Polynomial order 6, number of knots100

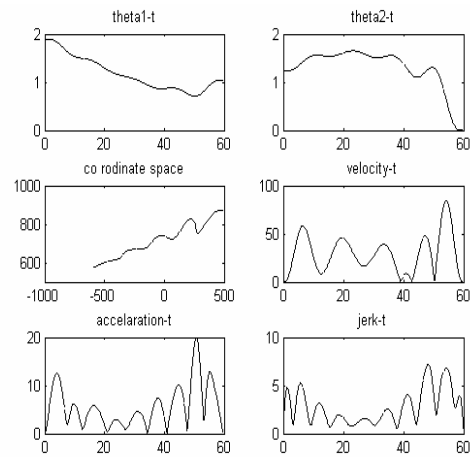


Fig 10. Polynomial order 8, number of knots10

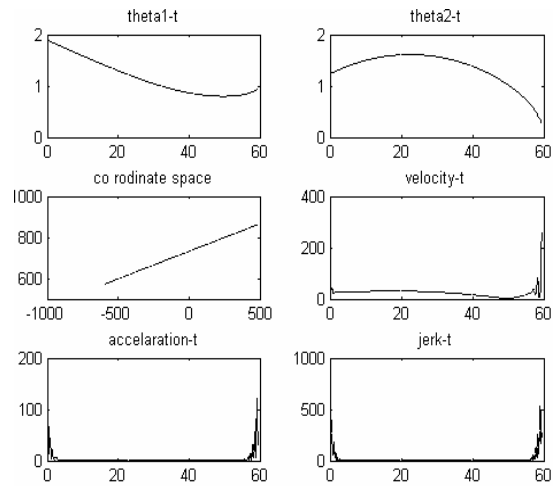


Fig 11. Polynomial order 8, number of knots100

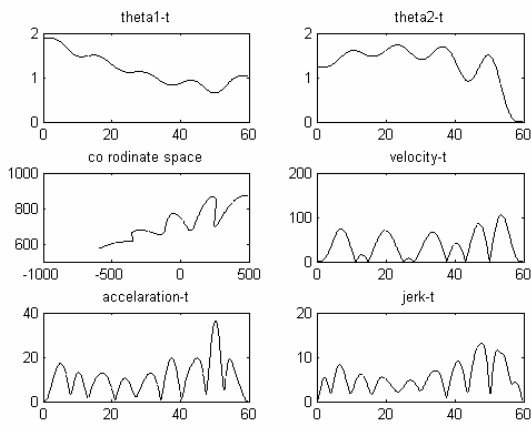


Fig.12. Polynomial order 10, number of knots10

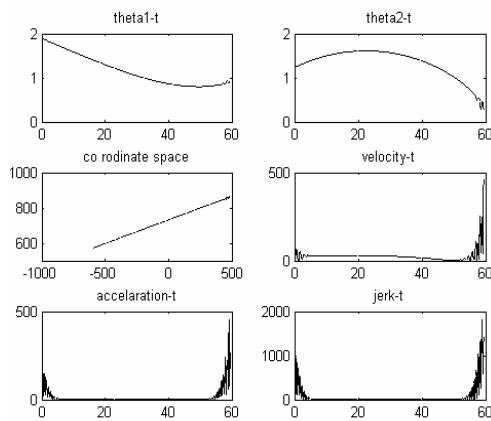


Fig.13. Polynomial order 10, number of knots100

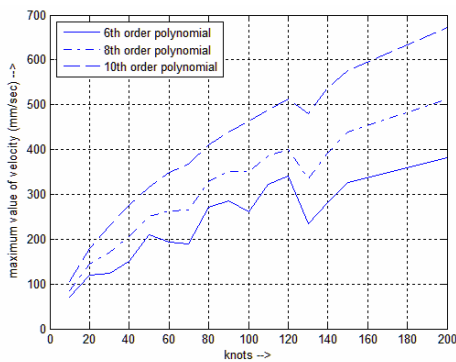


Fig 14. Max velocity vs. number of knots

From the above analysis the following observations are made:

For a particular order polynomial as we increase the number of knots (i) the trajectory approaches the target curve (ii) the velocity, acceleration and jerk have a very low and smooth value in the middle region, (iii) at the ends of completion of motion high fluctuations with

increased number of knots and (iv) maximum values of velocity, acceleration and jerk increase with the rise of number of knots.

And for a particular number knots as we vary the order of the polynomial,

(i) the duration of end fluctuation increases as we increase the order of polynomial; (ii) the values of maximum velocity, acceleration, jerk increase with the increase in order; (iii) as we increase the order of polynomial with number of knots the variation of maximum velocity, acceleration, jerk smoothens.

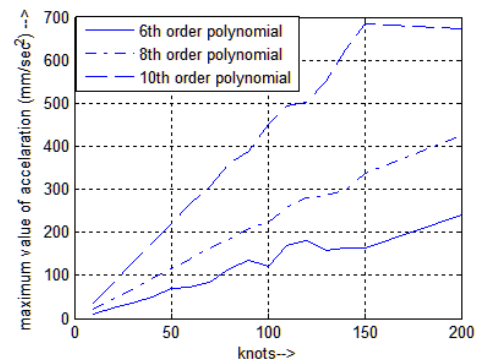


Fig 15. Max acceleration vs. number of knots

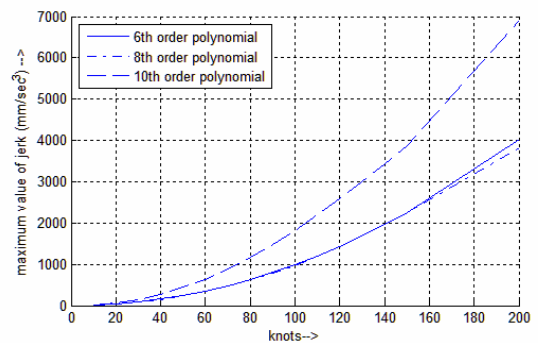


Fig 16. Max jerk vs. number of knots

## 5. CONCLUSION

The objective of the work was to study the effect of the value of the order of the polynomials and number knots on motion characteristics while designing trajectory for a manipulator. In this regard the effect of number of intermediate knots between two positions within the workspace has been studied for 6-, 8-, and 10-order polynomials.

From the above analysis it is evident that as we increase the number of knots the trajectory approaches the target curve and the middle region gives smooth and almost constant values of velocity, acceleration and jerk. Also with the rise of both value of order and number of knots we get smooth variation of those values. It is evident that there is a scope for optimization of peak values of jerk and acceleration considering variation of number of knot and value of the order of polynomial.

## 6. REFERENCES

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## 7. NOMENCLATURE

Symbol	Meaning	Unit
$\theta_1, \theta_2, \theta_3$	Joint angles.	(rad)
$c_{12\dots i}$	$\cos(\theta_1 + \theta_2 + \dots + \theta_i)$	
$s_{12\dots i}$	$\sin(\theta_1 + \theta_2 + \dots + \theta_i)$	
$L_1$	Length of first link of end-effector.	Mm
$L_2$	Length of second link of end-effector.	Mm
$L_3$	Length of third link of end-effector.	Mm
$V_R$	Resultant velocity of end-effector.	Mm/sec
$a_R$	Resultant acceleration of end-effector.	Mm/sec <sup>2</sup>
$\dot{J}_R$	Resultant jerk of end-effector.	Mm/sec <sup>3</sup>

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